On-Line Geometric Modeling Notes

BÉZIER CURVES

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Overview

The curve representation that is most frequently used in computer graphics, was independently discovered by Pierre Bézier (pronounced Bez-ye), who was an engineer for Renault and Paul de Casteljau, who was an engineer for Citroën, both working for automobile companies in France. These engineers developed a scheme that has both analytical and geometrical roots:the parameter values are points in three-dimensional space, which are blended together by polynomial functions. Being competitors, both companies were very secretive about their work and although De Casteljau's work was slightly earlier than Bézier's, it was never published; so the field retains Bézier's name. However, the fundamental algorithm which forms the basis for the constructions and calculation for Bézier curves is now credited to de Casteljau.

The design and analysis of curves and surfaces belongs to a field of study called geometric modeling, or computer-aided geometric design. Students of this field use mathematical techniques to design curve and surface representations that can be easily implemented on computer systems. The Bézier curve is the cornerstone of this field. The curve has a simple formulation – either through geometrical or analytical means – and is very useful in modeling and design. In these notes we examine the Bézier curve and the many ways that it can be described.

If you are a novice to this subject, it is suggested that you begin with the section that describes a divide-and-conquer strategy for generating a curve, which gives a simple introduction to the ideas behind this curve. You should then proceed via the discussions of the quadratic Bézier curve and the cubic Bézier curve which introduce the most commonly used Bézier curves and their properties.

- A Divide and Conquer Method for Drawing a Simple Quadratic Curve.
- The Bernstein Polynomials.

- The Quadratic Bézier Curve.
- The Cubic Bézier Curve
- The General Equation for a Bézier Curve of Arbitrary Order.
- A Matrix Formulation of the Cubic Bézier Curve.
