On-Line Geometric Modeling Notes

BÉZIER PATCH SUBDIVISION

Kenneth I. Joy
Visualization and Graphics Research Group
Department of Computer Science
University of California, Davis

Overview

A general method can be specified to subdivide a Bézier patch. This method is specified unlike the matrix methods, as it is based upon the definition of the patch as a set of curves.

The Method for Subdivision

We recall that, if we take the analytic equation of a Bézier patch, fix \( u \) and group factors appropriately, we obtain

\[
P(u, v) = \sum_{j=0}^{m} \left[ \sum_{i=0}^{n} P_{i,j} B_{i,n}(u) \right] B_{j,m}(v)
\]

We notice that portion of the equation inside the brackets is the representation of a Bézier curve. If we let \( Q_j(u) \) be the value inside the brackets, i.e.

\[
Q_j(u) = \sum_{i=0}^{n} P_{i,j} B_{i,n}(u)
\]

Then

\[
P(u, v) = \sum_{j=0}^{m} Q_j(u) B_{j,m}(v)
\]

That is, the quantities \( Q_j(u) \) form the control points of another Bézier curve, and together for all \( u \) and \( v \), they form the surface.
If, then, we subdivide each of the $m$ rows of the $P_{i,j}$ matrix, it implies that the $Q_j$s in the above equation represent only points from the first half of the patch (with respect to $u$). The following illustration shows the result of subdividing the rows in the $4 \times 4$ case.

The second half of the patch can be obtained in a similar fashion. The first and second half of the patch, with respect to $v$, can be obtained by subdividing the columns.

**Summary**

So, using only curve methods, and by subdividing the rows or columns of the control point array, we can effectively subdivide a Bézier patch. This is the most frequently used algorithm in software implementations of subdivision and can be utilized for Bézier patches of arbitrary degree.