

On-Line Geometric Modeling Notes

REPARAMETERIZING BÉZIER CURVES

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Overview

The Bézier curve is the representation that is most utilized in computer graphics and geometric modeling. This curve is usually defined by a set of control points $\{\mathbf{P}_0, \mathbf{P}_1, \dots, \mathbf{P}_n\}$ where

$$\mathbf{P}(t) = \sum_{i=0}^n \mathbf{P}_i B_{i,n}(t)$$

for $0 \leq t \leq 1$.

Running the parameter t from 0 to 1 gives a simple analytic and geometric definition of the curve. However, when we wish to examine general B-spline curves, which are piecewise Bézier curves, we will need to vary this parameter over an arbitrary interval. This is actually quite simple, and is discussed in the sections below.

Defining the Reparameterized Curve

Given a Bézier curve $\mathbf{P}(t)$, we can develop a new parameterization of the curve where t ranges between the values a and b by

$$\mathbf{P}_{[a,b]}(t) = \mathbf{P}\left(\frac{t-a}{b-a}\right)$$

We note that $\mathbf{P}_{[a,b]}$ and $\mathbf{P}(t)$ are exactly the same curve, but traversed through different ranges of t . This change impacts only a few of the Bézier curve properties, namely

- $\mathbf{P}_{[0,1]}(t) = \mathbf{P}(t)$.

- Using the chain rule, the derivative of the curve $\mathbf{P}_{[a,b]}(t)$ at a value t is equal to

$$\frac{1}{b-a} \mathbf{P}'\left(\frac{t-a}{b-a}\right)$$

- Subdividing the curve $\mathbf{P}_{[a,b]}(t)$ at the point $c \in [a, b]$, is equivalent to subdividing the curve $\mathbf{P}(t)$ at the point $\frac{c-a}{b-a}$.

Summary

The Bézier curve is normally developed by using a parameter that ranges between 0 and 1. By a simple modification, we can reparameterize the curve so that t can range between any two values a and b . The resulting curve algorithms for $\mathbf{P}_{[a,b]}(t)$ can all be related to the algorithms for the originally defined $\mathbf{P}(t)$.

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