

## On-Line Geometric Modeling Notes

# SUBDIVISION CURVES

Kenneth I. Joy  
Visualization and Graphics Research Group  
Department of Computer Science  
University of California, Davis

Curve generation methods are an important topic in computer graphics and geometric modeling. A new set of methods is now becoming popular which utilize a control polygon, as in the Bézier or B-spline case, but instead of using analytic methods to directly calculate points on the curve, these methods successively refine the control polygons into an sequence of control polygons that, in the limit, converge to a curve. By doing this, freedom from a closed-form mathematical expression is achieved, and a wide variety of curve types can be expressed. The curves are commonly called *subdivision* curves as the methods are based upon the binary subdivision of the uniform B-spline curves.

As it turns out, curves are fairly straightforward, and the interesting cases are surfaces and solids where the topology of the underlying control mesh can be quite complex. However, the curve cases are easier to present and therefore an understanding of these sections is important before proceeding to surfaces and solids. Most of the techniques presented here are similar to the techniques that are utilized in the surface cases.

To understand the basis of this method, the reader should first consult the notes on Chaikin's curve – which can be pointed to as the first of these algorithms.

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- What is a Subdivision/Refinement Process?
  - Chaikin's Curves - Curve generation by cutting corners.
  - Subdivision Curves based upon the Quadratic Uniform B-Spline Curve
  - Subdivision Curves based upon the Cubic Uniform B-Spline Curve
    - Classifying Points on the Subdivision as Vertex and Edge Points

- Direct Calculation of Points on Cubic Subdivision Curves
    - Rewriting the Refinement Procedure in Terms of a Matrix Equation
    - Calculating the Tangent Vectors at a Point
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