A Multi-resolution Data Structure for Two-dimensional Morse Functions

Peer-Timo Bremer[*‡] Herbert Edelsbrunner[†] Bernd Hamann[*] Valerio Pascucci[‡]

[*]Visualization and Graphics Research Group, University of California, Davis, CA 95616-8562
[†] Department of Computer Science, Duke University, Durham, NC 27708
Center for Applied Scientific Computing, Lawrence Livermore National Laboratory, Livermore, CA 94551

Abstract

The efficient construction of simplified models is a central problem in the field of visualization. We combine topological and geometric methods to construct a multi-resolution data structure for functions over two-dimensional domains. Starting with the Morse-Smale complex we build a hierarchy by progressively canceling critical points in pairs. The data structure supports mesh traversal operations similar to traditional multi-resolution representations.

1 Introduction

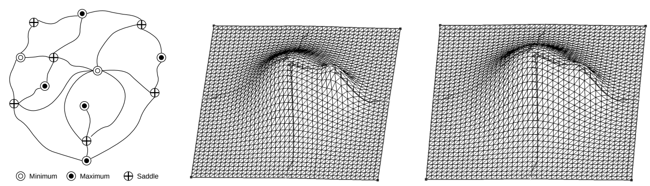
Scientific data typically consists of measurements over a geometric domain or space. We think of it as a discrete sample of a continuous function over the space. In this paper we are interested in the case in which the space is a compact 2-manifold. Examples are the sphere and the torus, and either one of them can be obtained by compactifying a simply connected open region of the plane.

A multi-resolution representation is crucial in the efficient and preferably interactive exploration of scientific data. Traditional approaches of constructing such hierarchies are based on progressive simplification guided by a numerical measurement of error. Alternatively, we may drive the simplification process with measurements of the topological features in the data. The latter approach is appropriate if the topological features and their spatial relationships are essential to understand the phenomena under investigation. An example is water flow over a terrain, which is influenced by possibly subtle slopes. Small but critical changes in the landscape may result in catastrophic changes in water flow and accumulation.

The topological analysis of scientific data has been a long standing research focus. Morse theory related methods were already developed in the late 19th century [1, 11] and later even hierarchical representations were proposed [12, 13]. Most modern research in the area of multi-resolution structures is geometry based and many techniques have been developed during the last decade [9, 14, 7, 10]. Subsequently, many of these algorithms have been extended to allow the topology type of the surface to change [6, 8]. In this paper we use a different approach. We directly compute the topology of the data (represented by its *Morse complex*) and use it to guide the construction of a hierarchy. The construction and simplification of Morse complexes is based on the work of Edelsbrunner et al. [3, 2] as well as the error metric we use during the simplification [4].

2 Background

We are concerned with a real-valued smooth function f over a compact 2-manifold M without boundary. The topology of f can be described by considering its *critical points* (point with zero derivative) and their relationships. There exist three types of critical points, minima, maxima and saddle points. Additionally, we consider *integral lines* which follow the local gradient of the function (lines of steepest ascent/descent). Ignoring special cases, each saddle is the intersection point of two integral lines, or the starting point of two lines of steepest ascent and two lines of steepest descent. By tracing out these lines we compute what is called a Morse complex, see Figure 1(a). A Morse complex consists of four-sided regions



(a) (b)

Figure 1: (a) A sample Morse complex including a degenerate cell. (b) An example of canceling a saddle and a maximum: (left) original surface; (right) simplified surface.

(*Morse cells*) each of which contain a minimum, a saddle, a maximum, and a saddle in order. Since inside each Morse cell the data is monotone, this completely describes the topology of f. A Morse complex can be simplified by canceling pairs of critical points, see Figure 1(b). Each cancellation removes a topological feature from the data. The error metric we use to rank cancellations is distance in function space between the two critical points that are canceled. This metric has recently been called *persistence* [4]. The cancellation together with persistence as error metric can now be used to create a hierarchical representation of a Morse complex. By approximating f within each Morse cell this also induces a geometric hierarchy for f. This hierarchy can be used to create a large number of different view-dependent approximations of f each of which is guaranteed to be topological correct and observes tight geometric error bounds.

3 Algorithm

We compute the Morse complex by constructing two lines of steepest ascent and two lines of steepest descent staring from each saddle. We follow the local gradient introducing new edges whenever the gradient flows through a triangle rather than along edges. Following some simple rules we avoid degenerate cases where the interior of a Morse cell does not touch both saddles or is not simply connected. We are careful in defining robust algorithms that alway produce consistent results. Especially in degenerate regions, where several vertices may have the same function value, the greedy choices of local steepest ascent/descent may not work consistently. We address this problem using a technique based on the *simulation of simplicity* [5]. Each edge is given a direction that is then used to break ties. Now, all vertices in the data set can be treated as if they were in general position.

We build the multi-resolution data structure from bottom to top. The bottom layer stores the original Morse complex. Higher layers are created by successively performing cancellations. Each node in the hierarchy symbolizes one cancellation and dependencies are created by examining the region of interference for each cancellation. The region of interference is defined as the region in which the function changes during a cancellation. If two regions of interference have a non-empty intersection the two corresponding cancellations are dependent and a link between their nodes is created in the hierarchy.

During cancellations, the geometry can be adapted using iterative smoothing operations. For each Morse cell tight error bounds for the geometry exist in which we can optimize the function. For rendering purposes we remesh each cell to a regular 4-8 mesh structure. This allows interactive rendering of the current approximation.

4 Results

We tested our algorithm on the Puget Sound terrain data set at resolution 1025-by-1025 and elevation values represented with two-byte unsigned integers. Several resolutions of the Puget Sound data set are shown in Figure 2. After removal of all topology with persistence below 0.5% of the maximum elevation the approximation has only 4045 critical points of the 49185 in the original topology (too dense to be shown). These points determine the highest-resolution MS complex. The middle figure shows an approximation with 2025 critical points and a uniform persistence of about 1.2%. On the bottom is a view-dependent adaptive refinement based on the purple view frustum, which yield an approximation with 1070 critical points. The full-resolution topology is preserved inside the frustum, while outside only the minimal dependent topology is maintained. Note how the topology can drop quickly from the highest to the lowest resolution while maintaining a consistent mesh.

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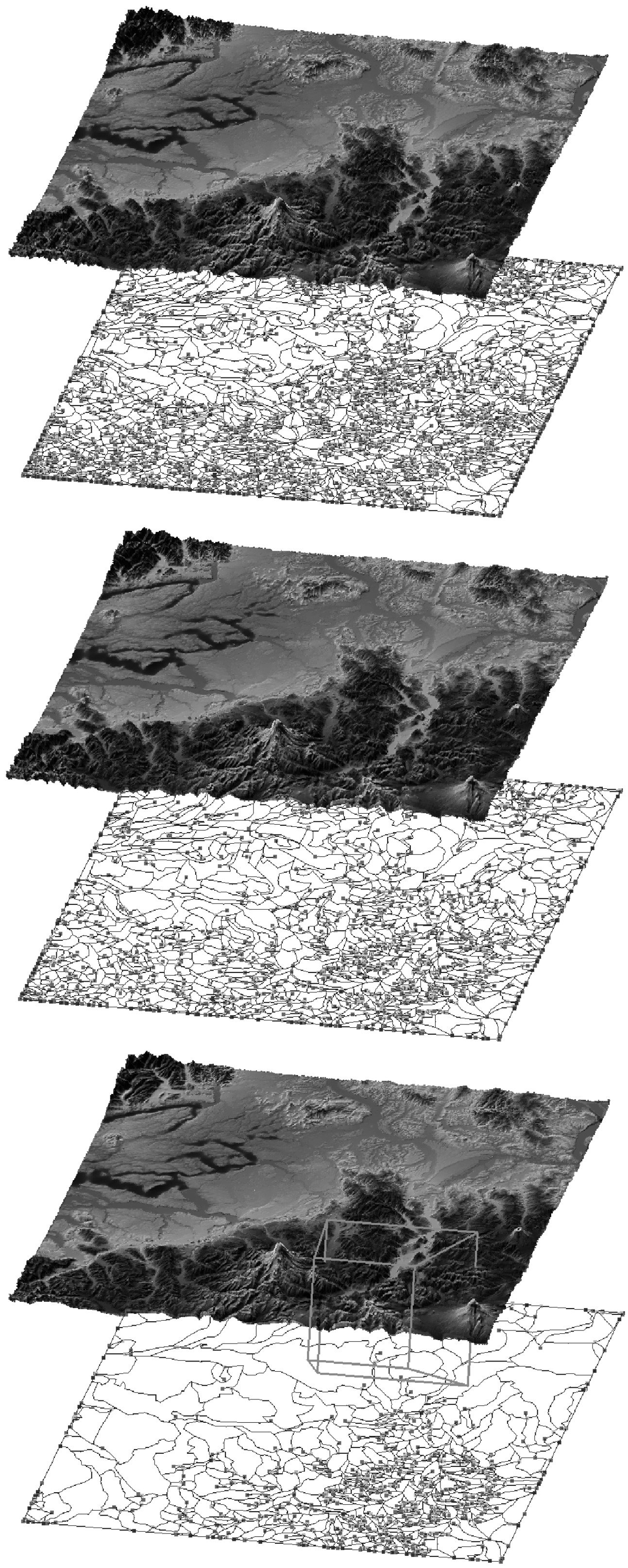

Figure 2: From top to bottom: Puget Sound data set after topological noise removal. A textured rendering is shown together with the corresponding MS complex; the data set at persistence of 1.2%; view-dependent refinement - current frustum shown in purple.