

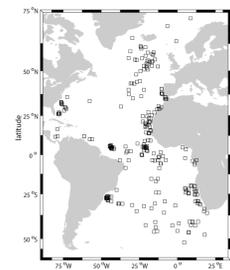
# A Comparison of Methods for Ocean Reconstruction from Sparse Observations



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## Introduction



- Problem: reconstructions from scattered observations of sediment cores distributed on the ocean floor in a sparse and irregular manner
- Data: measurements from benthic foraminifera in deep sea sediment cores (e.g., the data compiled by Peterson et al. [1])
- Solution: reconstruction methods useful for interpolating or approximating sparse scattered data
- Goal: comparison of the advantages and disadvantages of methods in order to enhance reconstruction quality

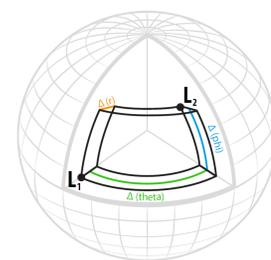
## Methods

### Reconstruction using a modified Moving Least Squares Approach

- Modified Moving Least Squares (MLS) approach without assuming any additional information (e.g., the flow field)
  - Input: A scalar field of observations  $\mathbf{f}$ , precomputed weighting parameters  $\alpha, \beta, \gamma$  (see Distance Measure section)
    - Computation of matrix  $\mathbf{B}$  from quadratic basis functions  $\mathbf{b}$
    - Set up diagonal weight matrix  $\mathbf{W}(\rho)$  with weight function  $\rho$  (depending on the weighted distance)
    - Evaluate MLS function<sup>[2]</sup>

$$MLS(\mathbf{x}) = \mathbf{b}^T(\mathbf{x})(\mathbf{B}^T \mathbf{W}(\rho) \mathbf{B})^{-1} \mathbf{B}^T \mathbf{W}(\rho) \mathbf{f},$$
 with  $\rho(L_1, L_2) = 1 / (\text{distance}(L_1, L_2)^2 + \epsilon^2)$   $\epsilon$  is a smoothing parameter
  - Output: Reconstructed values

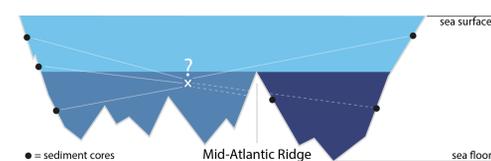
### Distance Measure



- Generalized distance measure considering spherical nature of the data set:
  - Input: Two locations  $\mathbf{L}_1$  and  $\mathbf{L}_2$ 

$$\text{distance}(L_1, L_2) = \alpha \cdot \Delta(\text{phi}) + \beta \cdot \Delta(\text{theta}) + \gamma \cdot \Delta(r)$$
    - $\alpha, \beta, \gamma$  are weights
    - $\Delta(\text{phi})$  and  $\Delta(\text{theta})$  are geodesic distances between  $\mathbf{L}_1$  and  $\mathbf{L}_2$  (see figure)
    - $\Delta(r)$  is the depth difference between  $\mathbf{L}_1$  and  $\mathbf{L}_2$
- Machine-learning pre-processing step performed to estimate good values for  $\alpha, \beta$  and  $\gamma$

### Physical Boundaries of the Ocean



- Consideration of ocean boundary to improve reconstruction results
- Core samples partitioned into bathymetry-based subsets
- Cores of not directly connected subsets are ignored

## Reconstruction using a Flow-Based Approach

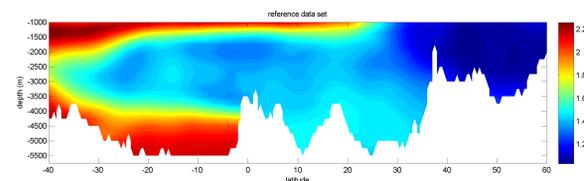
- Exploits correlations between the scalar field to be reconstructed and the vector field representing the ocean flow
- Input: A scalar field of observations  $\mathbf{f}$  and a vector field representing flow
- Output: Reconstructed values
- Optimal Interpolation used as the underlying reconstruction method
- Modification of underlying method: utilize a non-Euclidean distance measure defined using the input flow field:
 
$$\text{distance}(L_1, L_2) = \sqrt{(\alpha \cdot (\text{distance along streamline})^2 + (\text{distance across streamline})^2)}$$
- Streamlines are calculated for the flow field using a fourth-order Runge Kutta method
- Parameters:  $\alpha$ , correlation length
- Parameters optimized dynamically using an objective function defined with respect to the RMS error for a leave-one-out cross validation using the given observations

## Results

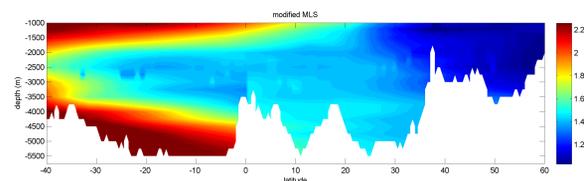
### Modern day

- Comparisons of the Reconstructions for the modern-day Atlantic Ocean based on a gridded data product [3] as reference data
- Reconstructions based on a subset of 186 data points corresponding to the distribution of the sediment cores
- Due to lack of a gridded climatology of modern-day  $\delta^{13}\text{C}$  data we used phosphate, because of its nearly linear relation to  $\delta^{13}\text{C}$

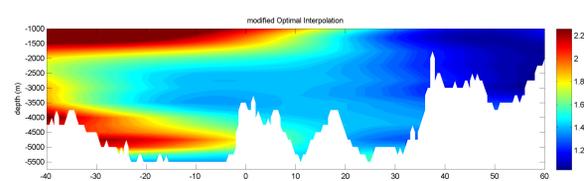
WOCE data set



Modified Moving Least Squares (MLS)

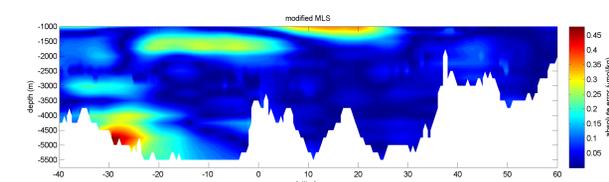


Modified Optimal Interpolation (OI)

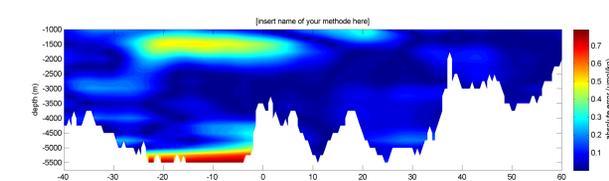


- Difference images between reconstructions and WOCE data set

Modified Moving Least Squares (MLS)

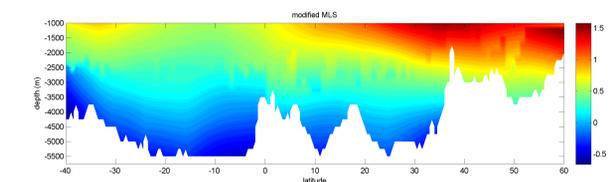


Modified Optimal Interpolation (OI)



## LGM

- A reconstruction for the LGM



## Summary and Future Work

- Both methods show promising results
- They have advantages and disadvantages in different regions
- In the future a combination of both may merged to a more precise reconstruction
- Further improvements may allow a estimating the glacial changes in multiple seawater properties and guide the analysis of existing and the future collection of sediment cores

## References

- Peterson, C., Lisiecki, L., and Stern, J. Deglacial whole-ocean  $\delta^{13}\text{C}$  change estimated from 493 benthic foraminiferal records. Submitted to *Paleoceanography*.
- Agranovsky, A., Garth C. and Joy, K., (2011) Extracting Flow Structures Using Sparse Particles. *Vision, Modeling and Visualization Workshop 2011*
- Gouretski, V., and K. Koltermann (2004), WOCE global hydrographic climatology, *Bericht des Bundesamtes für Seeschifffahrt und Hydrogr.*