Comparative Visual Analysis of Lagrangian Transport in CFD Ensembles

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Flow Field Ensembles

- Complex physical phenomena: repeated simulation runs with varying parameters
- Result: multiple time-varying fields sharing a spatial and temporal domain
- Focus of this work: agreement / disagreement with regard to transport
Flow Fields: Transport

• How is material transported throughout the flow domain?

• Extensively studied: divergence or convergence of particle neighborhoods

• Empirical study via discretized flow maps
  • e.g. Finite-Time Lyapunov Exponent (FTLE) [Haller 01]
Flow Map

- Discretize the domain
- Compute path lines for fixed time interval $T$, store endpoints → flow map
- Compute empirical measures from flow map samples
Flow Fields: Transport

• How to compare transport behavior of multiple vector fields?

• Our approach:
  • Compare flow maps of ensemble members
For each point of the domain:

- Flow map varies among runs
- Compute variance along first principal component (PCA) for particle positions: **Joint Variance**
Joint Variance

- Example: Time-varying simulation of convection around heated cylinder
- 30 simulation runs
- Small perturbation of flow at inflow boundary
- Small perturbation of initial flow field
In each run: Areas of high divergence

• Joint variance sensitive w.r.t. initial position
• Sensitivity with respect to slight perturbations: essentially same transport behavior, but high joint variance
Individual Variance

- Use a measure for intra-field divergence to assess reliability of joint variance interpretation

- Within each run, compute PCA for local neighborhood of every point: Individual Variance
Classification Space

![Diagram showing individual variance vs. joint variance with points labeled 0, 1, 2, 3, 4, 5.](image)
Classification Space
Classification Space

- Low average individual variance
- Low joint variance

- Particles move coherently both across the ensemble and within the neighborhood

- Reliable interpretation: agreement
Classification Space

- Low average individual variance
- High joint variance
- Large displacement variance across the ensemble
- Coherent particle motion within runs
- Reliable interpretation: disagreement
Classification Space

- High average individual variance
- High divergence in local neighborhoods
- Low joint variance suggests similar behavior
- High joint variance suggests dissimilar behavior
- No reliable interpretation available
Example: Convection

- Flow around heated cylinder
- 30 runs: slightly perturbed initial velocity at the bottom
- Discretization: 256x512
- Observed over 100 time steps
Visualization

- Define color map on classification space
- c.f. transfer functions for volume rendering
  [Kindlmann et al. 98]
  [Kniss et al. 02]
Example: Convection

- Modify thresholds for low and high variance values
Outliers and Trends
Industrial Stirring

- Two counter-rotating pairs of mixing rods in a cylindrical vessel
- Discretization: 336x336
- 20 runs: slightly varying viscosity
- Observed over 150 time steps
Conclusion

• Classification space: variance-based comparison technique for fluid transport in vector field ensembles (2D and 3D)
• Automatic detection of trends and outliers
• Interactive exploration

Future Work:
• Link trends and outliers to simulation parameters
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## Examples

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<th>Simulation</th>
<th>Resolution</th>
<th>Time steps</th>
<th>Runs</th>
<th>Flow maps (per run)</th>
<th>Flow maps (sum)</th>
<th>Variances</th>
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<tr>
<td>Convection 2D</td>
<td>256x512</td>
<td>100</td>
<td>30</td>
<td>23s</td>
<td>11min, 30s</td>
<td>15s</td>
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<td>Stirring</td>
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<td>20</td>
<td>9s</td>
<td>3min</td>
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<td>Rayleigh-Taylor</td>
<td>68x68x68</td>
<td>51</td>
<td>8</td>
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<td>3s</td>
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<tr>
<td>Convection 3D</td>
<td>64x128x64</td>
<td>11</td>
<td>10</td>
<td>12s</td>
<td>2min</td>
<td>7s</td>
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Why not FTLE instead of individual variance?

- Variances have a more clear geometric interpretation
- We want relate two variances to each other to construct the classification space.
- More general definition of neighborhoods in the paper: Larger neighborhoods do not lend themselves to FTLE computation
Why average individual variance?

- Multiple ways to collapse individual variances are possible
  - Average: represents average reliability
  - Maximum: represents minimal reliability

- Distribution of individual variance is likely influenced by presence of trends in the ensemble
- Future Work: more elaborate analysis of individual variance