Overview

A cubic Bézier patch has a useful representation when written in a matrix form. This form allows us to specify many operations with Bézier patches as matrix operations which can be performed quickly on computer systems optimized for geometry operations with matrices.

This is an unusual representation for many students as it is not frequently shown in basic courses. If you have not seen this before it is suggested that you begin with the section on matrix representations for Bézier curves in which the equations are simpler and easier to understand.

Developing the Matrix Formulation

We have seen that a cubic Bézier curve can be written in a convenient matrix form. Similarly, a bicubic Bézier patch can be written in a matrix form by using methods similar to that for a Bézier
Curve. Utilizing the representation of a Bézier patch as a continuous set of Bézier curves, we have

\[ P(u, v) = \sum_{j=0}^{3} \sum_{i=0}^{3} P_{i,j} B_{i,3}(u) B_{j,3}(v) \]

\[ = \sum_{j=0}^{3} \left[ \sum_{i=0}^{3} P_{i,j} B_{i,3}(u) \right] B_{j,3}(v) \]

\[ = \sum_{j=0}^{3} \left[ 1 \ u \ u^2 \ u^3 \right] \left[ \begin{array}{cccc} 1 & 0 & 0 & 0 \\ -3 & 3 & 0 & 0 \\ 3 & -6 & 3 & 0 \\ -1 & 3 & -3 & 1 \end{array} \right] \begin{bmatrix} P_{0,j} \\ P_{1,j} \\ P_{2,j} \\ P_{3,j} \end{bmatrix} B_{j,3}(v) \]

and so the cubic Bézier patch is frequently written

\[ P(u, v) = \left[ 1 \ u \ u^2 \ u^3 \right] M \begin{bmatrix} P_{0,0} & P_{0,1} & P_{0,2} & P_{0,3} \\ P_{1,0} & P_{1,1} & P_{1,2} & P_{1,3} \\ P_{2,0} & P_{2,1} & P_{2,2} & P_{2,3} \\ P_{3,0} & P_{3,1} & P_{3,2} & P_{3,3} \end{bmatrix} M^T \begin{bmatrix} 1 \\ v \\ v^2 \\ v^3 \end{bmatrix} \]

where

\[ M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 3 & 0 & 0 \\ 3 & -6 & 3 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix} \]

Summary

We have developed a matrix form for the Bézier patch which parallels the development for the Bézier curve. These matrix equations exist for patches of all orders – we have done order 4 (or degree 3) patches here. However, the matrices are \( n \times n \) for a patch of order \( n \).