

PERCEPTUAL COLOR SPACES FOR COMPUTER GRAPHICS

by

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ABSTRACT

Perceptually uniform color spaces can be a useful tool for solving computer graphics color selection problems. However, before they can be used effectively some basic principles of tristimulus colorimetry must be understood and the color reproduction device on which they are to be used must be properly adjusted. The Munsell Book of Color and the Optical Society of America (OSA) Uniform Color Scale are two uniform color spaces which provide a useful way of organizing the colors of a digitally controlled color television monitor. The perceptual uniformity of these color spaces can be used to select color scales to encode the variations of parameters such as temperature or stress.

COMPUTING REVIEWS CLASSIFICATION: 3.1, 3.2, 3.41, 8.2

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1. INTRODUCTION

A perceptually uniform color organization has been sought for years by color scientists attempting to set tolerances on color reproduction techniques, by psychologists probing the psychophysiology of vision, and by artists looking for new color harmonies. The idea is to define a color system in which an equal perceptual distance separates all of the colors. For example, the grayscale of the system should provide a smooth transition between black and white. Although such an ideal system has yet to be found, numerous proposals have been made for approximately uniform systems. Most of these proposed color organizations are described in terms of the color notation system standardized by the Commission Internationale de L'Eclairage (CIE).

Since color television is based on the CIE color notation system, these uniform spaces can be directly applied to work with digitally controlled color television monitors once the colorimetry and calibration of the monitors is understood. Color collections such as the Munsell Book of Color and the Optical Society of America (OSA) Uniform Color Scale can supplement the color organizations described in JOBL78 and SMIT78 as palettes from which

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to make color selections. In addition the perceptual uniformity of these color spaces can be used in such problems as color encoding of information and image data compression.

2. COLOR SCIENCE AND TELEVISION COLORIMETRY

Only the fundamental results of tri-chromatic color theory and television colorimetry will be presented here. For more detailed information, the reader is referred to HUNT75, JUDD75, and WYSZ67 which are excellent general references on color science and color reproduction. Television colorimetry is discussed in NEAL73, WENT55, and WINT51.

A basic assumption in tri-stimulus colorimetry is that the perception of color is primarily determined by the spectral energy distribution of the electromagnetic energy entering the eye. Experiment has shown that the color sensation produced by a given spectral energy distribution can be quantified as a triplet of numbers computed from the expressions:

$$X = \int_{380}^{780} E(\lambda) \bar{x}(\lambda) d\lambda \quad (1)$$

$$Y = \int_{380}^{780} E(\lambda) \bar{y}(\lambda) d\lambda$$

$$Z = \int_{380}^{780} E(\lambda) \bar{z}(\lambda) d\lambda$$

where X, Y, and Z are referred to as the tristimulus values of the color, $E(\lambda)$ is the spectral energy distribution of the color and $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, and

$\bar{z}(\lambda)$ are the color matching functions (Figure 1). The Y tristimulus value is referred to as the luminance of the color and is expressed either as an absolute luminance in candelas per square meter or as a percent luminous reflectance.

When plotted, the locus of possible tristimulus values is found to be bounded by a cone called the cone of realizable color (Figure 2). This coordinate system is referred to as 1931 CIE XYZ space. Given that spectral energy distributions of identical shape but different relative height (i.e., a variable intensity light source) arise frequently and that the locus of tristimulus values for these distributions is a straight line through the origin of XYZ space, it is often useful to adopt an alternative notation system in which a color is described by the direction and magnitude of a vector through the origin. The point of intersection of the vector with the unit plane in 1931 CIE XYZ space expresses the direction of the vector and is found from the tristimulus values by:

$$x = \frac{X}{X + Y + Z} \quad (2)$$

$$y = \frac{Y}{X + Y + Z}$$

Once the direction of the vector is established, one tristimulus value (typically Y) is all that is required to define the vector's magnitude. Two-dimensional plots of x versus y are known as chromaticity diagrams.

The tristimulus values can be transformed into a new coordinate system with origin coincident to that of 1931 CIE XYZ space by an expression of the form:

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} X_R & X_G & X_B \\ Y_R & Y_G & Y_B \\ Z_R & Z_G & Z_B \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \quad (3)$$

The terms of the matrix can be found from the orientation of the new coordinate system's axes relative to the 1931 CIE XYZ space axes and from the coordinates of one point in both the new system and 1931 CIE XYZ space. Furthermore, tri-chromatic color theory states that if the axes of the new coordinate system correspond with the loci for three variable intensity light sources (see above discussion of chromaticity coordinates) then the resultant spectral energy distribution obtained by mixing R units of the first light, G units of the second light, and B units of the third light will be perceived as having the same color as the spectral energy distribution which yielded (via equation (1)) the original XYZ tristimulus values even though the two spectral energy distributions may be different.

For color television the directions of the axes for the RGB space of equation (3) are determined by the chromaticity coordinates of the television monitor's phosphors. The common point between the 1931 CIE XYZ system and RGB space necessary to fully specify the transformation is re-

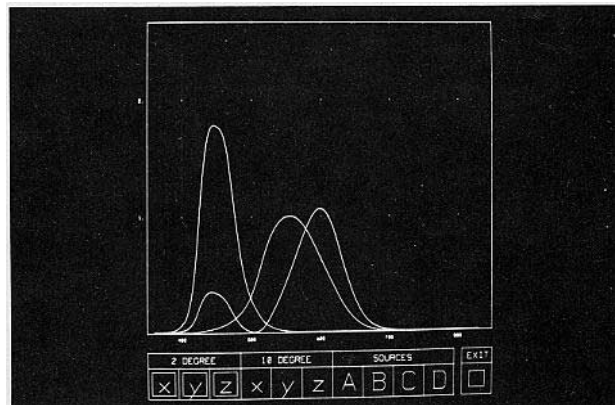


Figure 1: From left to right, the color matching functions $\bar{z}(\lambda)$, $\bar{y}(\lambda)$, $\bar{x}(\lambda)$ (tristimulus values of equal-energy spectrum) for the 1931 standard observer.

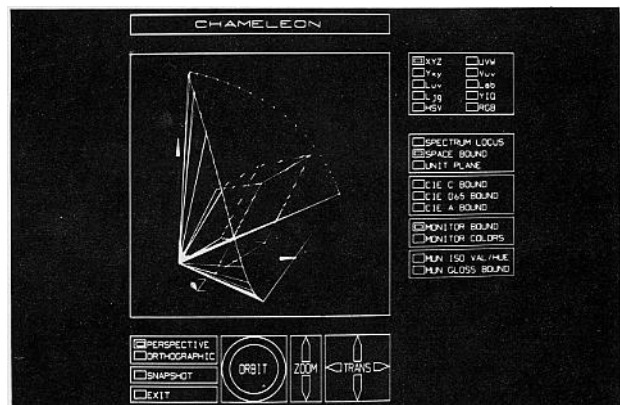


Figure 2: Cone of realizable color (drawn to an arbitrary length) and typical color television monitor gamut in 1931 CIE XYZ space.

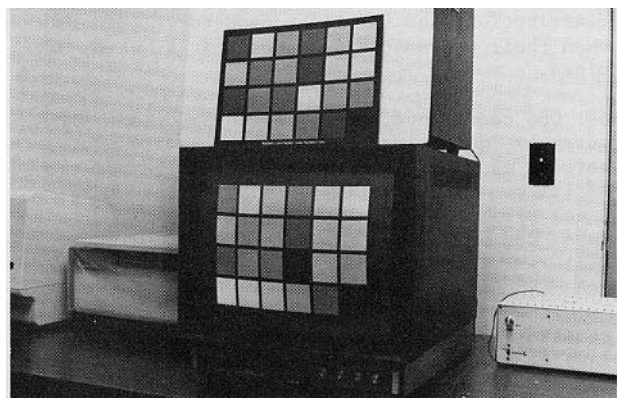


Figure 3: Comparison of color samples with monitor reproduction. Note that color in the third row and fourth column of the chart lies outside this monitor's gamut. (Photographic process has distorted relative brightness of chart and monitor colors).

ferred to as the "white point". Its coordinates in 1931 CIE XYZ space are usually given in terms of chromaticity coordinates and luminance. The monitor is adjusted so that this color has the coordinates in RGB space of the maximum RGB values. (In this paper the RGB values lie on the range 0.0 to 1.0). The mathematical relationship between the elements of the matrix in equation (3) and the above monitor properties can be found in several sources (MEYE80, NEAL73, WINT51) and will not be repeated here. Figure 2 shows the shape and position of a typical color television monitor gamut in 1931 CIE XYZ space.

In a typical color television monitor, the RGB values are controlled indirectly by specifying signal voltages R_{volt} , G_{volt} , and B_{volt} , normalized to the range 0.0 to 1.0 in this paper). The relation between, for example, R and R_{volt} can be determined experimentally by varying R_{volt} and using a digital photometer to measure the luminance (Y) produced at the display. This is simply a variable intensity light source of known chromaticity and the procedure generates points which define the R axis in 1931 CIE XYZ space. When repeated for the G and B signals, the following relations are established:

$$\begin{aligned} R_{\text{volt}} &= R \frac{1}{\gamma_R} \\ G_{\text{volt}} &= G \frac{1}{\gamma_G} \\ B_{\text{volt}} &= B \frac{1}{\gamma_B} \end{aligned} \quad (4)$$

where γ_R , γ_G , and γ_B are almost identical (often they are assumed identical and referred to collectively as the monitor "gamma") and lie on the range 2.5 to 3.0, depending on the monitor.

The validity of the transformation given by equations (3) and (4) can be tested by reproducing colors of known chromaticity and relative luminance and comparing them to actual color samples. Figure 3 shows such an experiment. In making these comparisons it is important that the color samples are illuminated with light of the same spectral energy distribution as the light used to illuminate them when their chromaticity coordinates were determined.

One cannot overemphasize the importance of accurately measuring and adjusting such monitor properties as phosphor chromaticity, white point chromaticity, white point luminance and gamma. If these parameters are not tightly controlled, accurate and consistent color reproduction is impossible (MEYE80).

It must also be remembered that variables held constant during the experiments which led to the laws of tristimulus colorimetry cannot be allowed to vary when these laws are applied. The level of ambient illumination has a significant effect on color perception and is something which can be quite different for an observer viewing an original scene and an observer viewing a reproduction of that scene on a monitor in a dimly lit room. Adjustments to straight colorimetric calculations

to account for this have been suggested and are discussed elsewhere (BART67, DEMA72, MEYE80, NOVI69).

3. MUNSELL SYSTEM

In 1905 Albert H. Munsell published a book and a series of color charts which proposed a color notation system with three dimensions: Hue, Value, and Chroma. His definitions for these terms were (MUNS46):

Hue - "It is that quality by which we distinguish one color family from another, as red from yellow, or green from blue or purple."

Value - "It is that quality by which we distinguish a light color from a dark one."

Chroma - "It is that quality of color by which we distinguish a strong color from a weak one; the degree of departure of a color sensation from that of white or gray; the intensity of a distinctive Hue; color intensity."

Although Munsell used a sphere in his original publication to describe the geometry of his color coordinate system, it is more consistent with the organization of the Munsell Book of Color (MACB79) to think of the notation system in cylindrical coordinates with hue as the angle relative to the cylinder's central axis, value the vertical position, and chroma the radial position.

In addition to labeling colors using hue, value, and chroma, Munsell also wanted a notation system which demonstrated his ideas about color balance. For example, colors which lie at opposite sides of the color solid are "balanced" about the central neutral gray. This example and others like it suggest that there is a psychological nature to his color system, i.e., steps along the hue, value, or chroma directions are perceptually equal.

In 1940 the CIE tristimulus values for the Munsell Book of Color were measured in terms of CIE illuminant C, and a new study of their perceptual spacing was undertaken. Forty observers made some 3,000,000 color judgements which resulted in a re-designation of the hue, value, and chroma specification for each color sample (NEWH40). Using this data, the relation between a sample's percent luminous reflectance (Y) and value Λ was found to be (NEWH43):

$$Y = 1.2219\Lambda - 0.23111\Lambda^2 + 0.23951\Lambda^3 - 0.021009\Lambda^4 + 0.0008404\Lambda^5 \quad (5)$$

The data for the samples was also plotted on chromaticity diagrams of constant value (and constant luminous reflectance according to equation (5)). Smooth curves defining loci of constant hue and constant chroma were drawn on the charts based on the redesignated Munsell samples (NEWH43). The results for value 5/ are shown in Figure 4. These loci were extrapolated to the 1/ and 9/ value levels and to the boundary of the object color solid

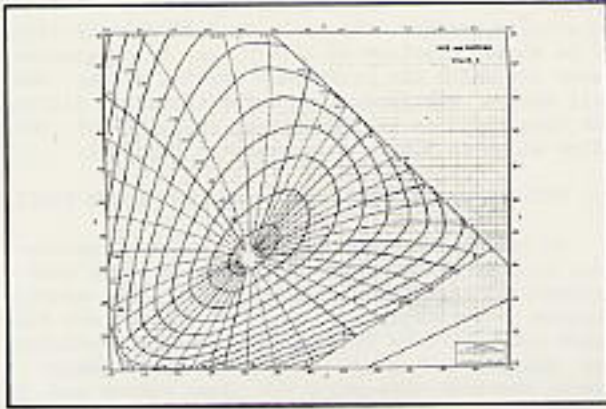


Figure 4: 1931 CIE chromaticity diagram showing loci of constant Munsell hue and chroma for Munsell colors of constant value 5/ (from JUDD75).

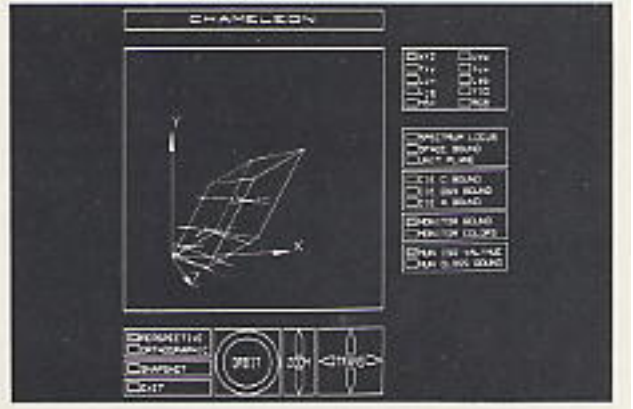


Figure 5: "Spiders" of constant Munsell value in 1931 CIE XYZ space and their position relative to the monitor gamut.

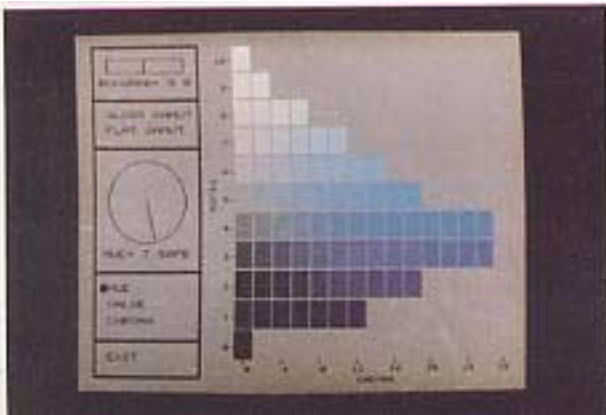


Figure 6: Constant hue slice through Munsell color solid.

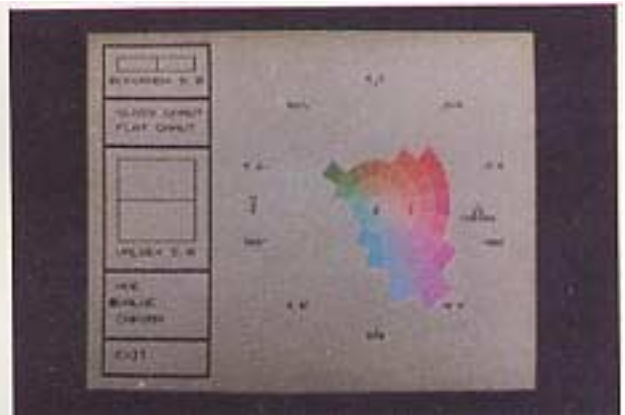


Figure 7: Constant value slice through Munsell color solid.

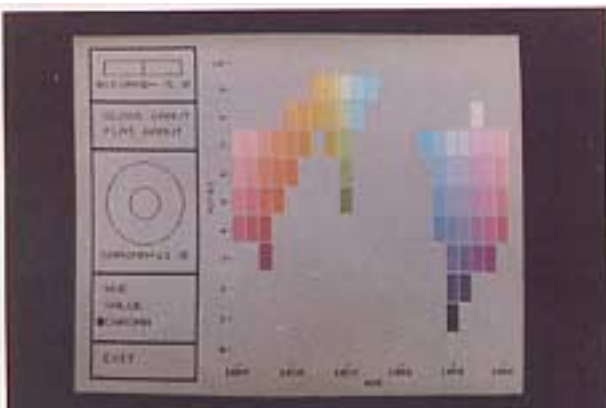


Figure 8: Constant chroma slice through Munsell color solid.

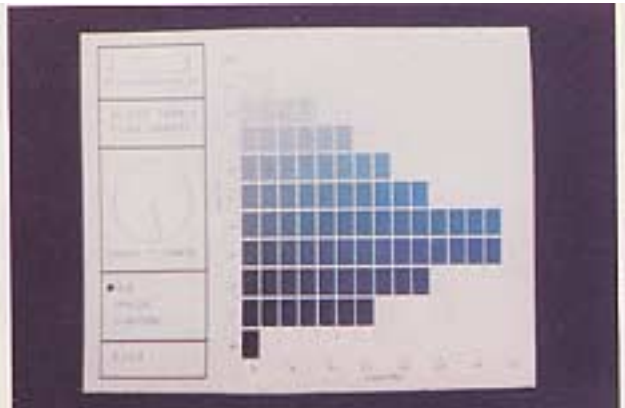


Figure 9: Hue slice of Figure 6 on background of Munsell value 10/.

for CIE illuminant C (MACA35). In 1954 these extrapolations were extended to the .2/ value level (JUDD56).

If the intersection points of the loci shown in Figure 4 are entered into a computer for all Munsell value levels, then the percent luminous reflectance for any Munsell color can be calculated from equation (5), and the chromaticity determined by suitable interpolation techniques from the chart data. Furthermore the color can be reproduced on a digitally controlled color television monitor by using equations (3) and (4). The percent luminous reflectance is converted to an absolute monitor luminance by using the percent luminous reflectance as a fraction of the maximum luminance with which the monitor can reproduce CIE illuminant C. This maximum luminance can be found by calculating where a color vector with chromaticity coordinates identical to CIE illuminant C intersects the monitor gamut in CIE XYZ space. With the absolute luminance and chromaticity coordinates, tristimulus values XYZ can be found and equations (3) and (4) applied to find the signal voltages necessary to reproduce the color on the monitor. Figure 5 shows "spiders" of constant Munsell value and their position within a typical color television monitor gamut. Each spider arm has constant hue, and the dots on the arm represent equal chroma spacings.

Deciding which Munsell renotation colors are reproducible on the monitor is difficult because the monitor and Munsell color gamuts have irregular shapes and their intersection is not well defined. The monitor gamut as shown in Figure 5 is determined by the phosphor chromaticities, the white point chromaticities, and the white point luminance. The Munsell renotation colors are bounded by the object-color solid (or theoretical pigment limit) as determined for CIE illuminant C (MACA35). It is of interest to note that the gamut of currently available pigment technology is not only smaller than the boundary of the object color solid but is also smaller than the gamut of color television. Hence, a color television monitor can produce Munsell renotation colors which have heretofore only been defined by extrapolation.

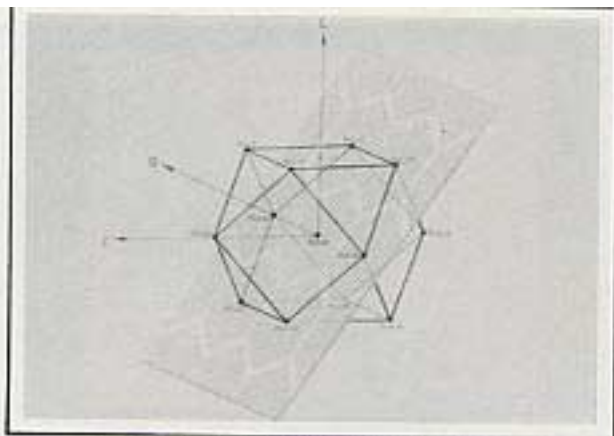


Figure 10: Cubo-octahedron which forms basic element of regular rhombohedral lattice. Vertices are labeled with their (L,j,g) notation. Cutting plane depicted corresponds to plane shown in Figure 11.

Figures 6, 7, and 8 show three slices through Munsell renotation space as produced on a color television monitor. The background of Munsell value 5/ in these displays is within the 5/ or greater range for which the perceptual spacing of the Munsell system was determined. The effect of different background on the perceptual spacing of the slice shown in Figure 6 is shown in Figure 9.

4. OPTICAL SOCIETY OF AMERICA UNIFORM COLOR SCALE

An inherent problem with any color organization such as the Munsell Book of Color which uses a cylindrical coordinate system is that the spacing between colors changes as two radial lines are followed outwards from the center of the cylinder. For distances between nearest color neighbors to remain constant throughout the color system and to maximize the number of equidistant nearest color neighbors for each color, a space lattice must be used. This is the approach taken in the Optical Society of America (OSA) Uniform Color Scale.

The basic element of the space lattice used in this system is the cubo-octahedron shown in Figure 10. A point at the center of the cubo-octahedron has the vertices of the polyhedron as 12 equidistant nearest neighbors. When these polyhedrons are packed together the result is called a regular rhombohedral lattice. Any point in the lattice is a member of six different linear color scales and seven different color planes.

The research which led to the specification of colors for each lattice point extended over some 30 years. "Judgements of relative magnitudes of color differences exhibited by 128 selected nearest-neighbor pairs of 59 colored tiles were recorded by 49 to 76 observers, all of whom had normal color vision (MACA74)." Forty-three of the colors had the same luminous reflectance and uniform chromaticity spacing according to available experimental results. Sixteen of the colors were arranged into four sets of four tiles, each set forming a regular tetrahedron in different portions of the tentative color space. The color specification which resulted from this data is (MACA74):

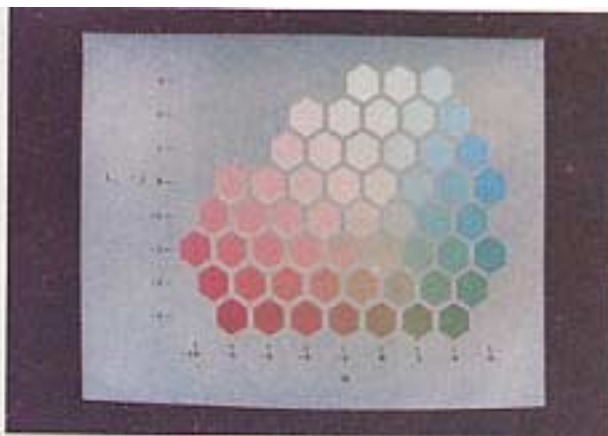


Figure 11: Monitor reproduction of the $L+j=0$ plane in the OSA committee uniform color space.

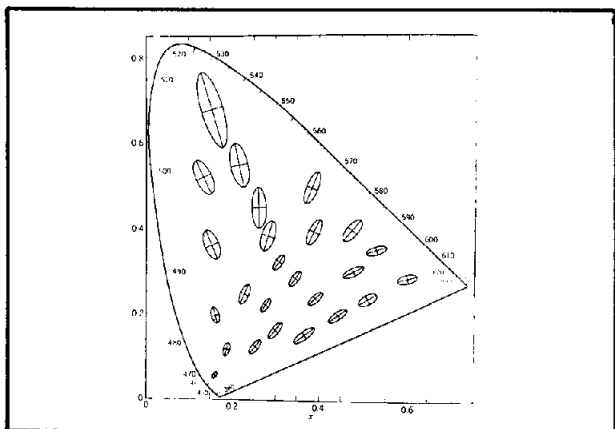


Figure 12: Statistical variation of chromaticity matches in different parts of the 1931 CIE chromaticity diagram. For clarity the axes of each ellipse have been enlarged 10 times (from JUDD75).

$$L = 5.9 [Y_o^{1/3} - 2/3 + 0.042(Y_o - 30)^{1/3}] \quad (6a)$$

$$Y_o = Y (4.4934x^2 + 4.3034y^2 - 4.276xy - 1.3744x - 2.5643y + 1.8103) \quad (6b)$$

$$g = C(-13.7R^{1/3} + 17.7G^{1/3} - 4B^{1/3}) \quad (6c)$$

$$j = C(1.7R^{1/3} + 8G^{1/3} - 9.7B^{1/3})$$

$$C = 1 + \frac{0.042(Y_o - 30)^{1/3}}{(Y_o^{1/3} - 2/3)} \quad (6d)$$

$$R = 0.799X + 0.4194Y - 0.1648Z \quad (6e)$$

$$G = -0.4493X + 1.3265Y + 0.0927Z$$

$$B = -0.1149X + 0.3394Y + 0.717Z$$

where L, j, and g stand respectively for lightness, yellowness (jaune in French), and greenness. X, Y, Z are the tristimulus values of the color (Y being expressed as percent luminous reflectance), and x, y are the color's chromaticity coordinates. They are based on D6500 illumination and the CIE 1964 supplementary observer. The correspondence between the L, j, g coordinates and the vertices of a cubo-octahedron is shown in Figure 10. The inverse of this transformation can be accomplished by an iterative numerical technique (MACA79). The formulae are valid only when the colors are viewed on a background of 30% reflectance gray and when an L, j, g triplet matches one of the approximately 558 triplets for which the transformation is defined (MACA78).

This color space can be reproduced on a color television monitor by use of equations (3) and (4). It is important to remember, however, that equation (6) is based on the CIE 1964 supplementary observer. This means that the phosphor chromaticity coordinates used to derive the matrix in equation (3) must be experimentally determined (MEYE80) since the chromaticity coordinates available from the manufacturer are based on the CIE 1931 standard observer. Figure 11 shows a monitor reproduction of a color plane from the OSA Uniform Color Scale.



Figure 13: Full color RGB image digitally transformed into YIQ color space and retransformed back into RGB space using only Y, only YI, and only YQ.

5. OTHER UNIFORM COLOR ORGANIZATIONS

Numerous other uniform color organizations have been proposed. Several that have gained wide acceptance are mentioned here.

Experiment has shown that the chromaticity diagram for 1931 CIE XYZ space does not have perceptually uniform spacing (Figure 12). A linear transformation has been proposed from 1931 CIE XYZ space to 1976 CIE UVW space so that the chromaticity diagram of this new space has a more perceptually uniform spacing (CIE78). A nonlinear transformation of the 1931 CIE chromaticity coordinates which directly yields the coordinates of a uniform chromaticity diagram has also been proposed (MACA71).

Two other uniform color spaces recommended by the CIE are designated CIE 1976 ($L^* u^* v^*$) and CIE 1976 ($L^* a^* b^*$) (CIE78). In both spaces two of the dimensions are coordinates on a uniform chromaticity diagram. In CIE 1976 ($L^* u^* v^*$) space the chromaticity diagram used is the one from 1976 CIE UVW space. The uniform chromaticity diagram used in CIE 1976 ($L^* a^* b^*$) space involves a nonlinear transformation from 1931 CIE XYZ space and is based on the cube-root version of the Adams-Nickerson color difference formula. The third dimension is identical in both spaces and is a simplification of equation (5) for Munsell value.

6. APPLICATION OF UNIFORM COLOR SPACES

The application of uniform color spaces to computer graphics problems must be prefaced with some words of caution. The data on which these uniform spaces are based was obtained under tightly controlled environmental conditions. Some of the parameters held constant include 1) the size of the color samples, 2) the spacing between color samples, 3) the luminance and chromaticity of the background on which the color samples were compared, and 4) the luminance and chromaticity of ambient light in the test environment. To maintain the perceptual uniformity of the space, the settings of these parameters in the final application must match the conditions existing when the data

was originally recorded. However, even with this limitation, these uniform spaces are still the best available tools for addressing the problems mentioned below.

When storing or transmitting color images, one should limit the information to no more than the viewer can perceive. Uniform color spaces can be used to decide what level of resolution the color information should be encoded at (LIMB77). The National Television System Committee (NTSC) color television standards incorporate a data compression scheme which has a perceptual basis but which was not derived using uniform color spaces (FINK55). The standard's YIQ transmission primaries are allocated bandwidths based on their relative importance in creating a subjectively acceptable color image. Reconstruction of a full color image from various combinations of the YIQ signal components is shown in Figure 13.

"False" coloring occurs when the true colors of an image are mapped into another set of colors. Black and white television, which generates images using only Y of the YIQ signal (Figure 13), is an example of each color being mapped to a position on a grayscale. While this is an effective data compression scheme, it makes it impossible to differentiate between colors with the same Y value. Often the intent of false coloring is to draw the observer's attention to certain features in an image by maximizing the difference in color between the feature and the surrounding area of the image. Uniform color spaces can be used to map image colors into a set with maximum perceptual spacing (BOOT77, DOUC77).

Color has also been used to encode the variations of parameters such as temperature or stress. The accuracy with which an observer interprets these images has been questioned (BOOT77, MORS79), but they still remain an effective means for showing data trends. For two-dimensional data plots uniform color spaces such as Munsell or OSA can be used to select color scales. Although this application of uniform color spaces violates many of the restrictions about size and spacing of color

samples mentioned at the beginning of this section, the results can still be quite satisfactory. Figures 14 and 15 show data being displayed using two different paths through the Munsell color solid. A problem in selecting such color scales from the Munsell and OSA color spaces is that the pigment gamut used to derive these spaces is generally smaller than the gamut of color television and the intersection of the television and pigment gamuts is irregular. This makes it difficult to find color scales which incorporate the most brilliant monitor colors and which lie completely within the area of intersection of the monitor and pigment gamuts. For three dimensional data presentations only chromaticity can be used to encode the parameter since the length of the color vector in 1931 CIE XYZ space is used for color intensity variations which convey the shape of the object. A uniform chromaticity diagram can be used to select a scale of chromaticity variations with uniform perceptual spacing.

7. SUMMARY

Uniform color spaces can be a useful tool for solving computer graphics color selection problems. This paper has presented some relevant uniform color spaces from the color science literature and has shown how these spaces can be applied to computer graphics work with digitally controlled color television monitors. In particular, the Munsell and OSA color organizations were reproduced as new palettes from which to make color selections, and a stress range was encoded using color scales selected from the Munsell color system. Uniform color spaces were also suggested as solutions to problems in image data compression and false coloring.

Although this paper was primarily concerned with existing uniform color spaces, future uniform color spaces will undoubtedly be defined with the aid of digitally controlled color television monitors. The flexibility and wide color gamut of this medium offer significant advantages over the complex formulae and limited color gamut which characterize the pigment technology used in the definition of current uniform color spaces.

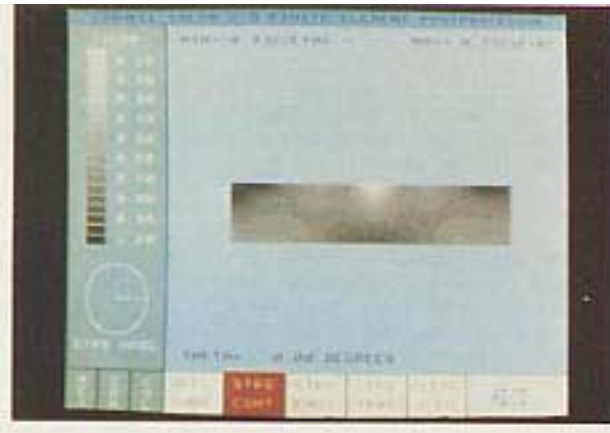


Figure 14: A stress variation from compression (top of color scale) to tension (bottom of color scale) encoded as a variation in Munsell value. Color scale goes from 10RP 9/4 (Hue Value/Chroma) at the top to 10RP 2/4 at the bottom.

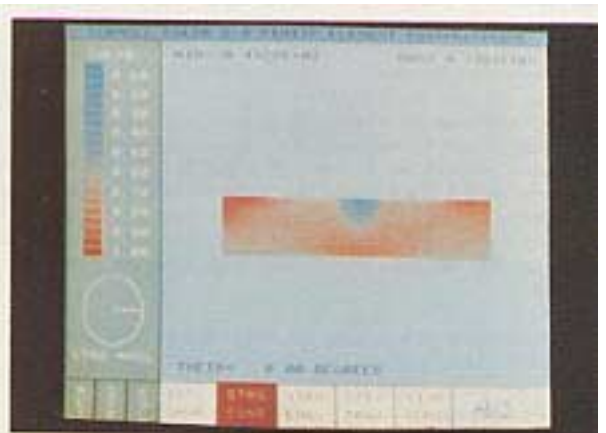


Figure 15: Alternative encoding for Figure 14. Top half of color scale goes from 5PB 5/12 to the neutral color 5PB 5.5/0. Bottom half of scale goes from this same neutral color (which has equivalent designation 7.5R 5.5/0) to the color 7.5R 6/18.

8. ACKNOWLEDGEMENTS

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